Exercise 24

For the following exercises, use the Intermediate Value Theorem to confirm that the given polynomial has at least one zero within the given interval.

$$f(x) = x^3 - 9x, \text{ between } x = -4 \text{ and } x = -2.$$

Solution

Plug $x = -4$ and $x = -2$ into the function.

$$f(-4) = (-4)^3 - 9(-4) = -28$$

$$f(-2) = (-2)^3 - 9(-2) = 10$$

Since $f(x)$ is a polynomial function (a smooth and continuous function), $f(x)$ has to take on every value between -28 and 10 for $-4 < x < -2$ by the Intermediate Value Theorem. Therefore, $f(x)$ has a zero between $x = -4$ and $x = -2$.